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Raman scattering in two-band superconductors

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Abstract. The electron Raman scattering in a two-band superconductor has been theoretically investigated. The fluctuational effects, Coulomb interaction, and electron scattering by non-magnetic impurities are taken into account. Two contributions to the light scattering intensity are found: an additive contribution from two bands, and an inter-band term which exists for arbitrary light polarization.

1. Introduction

The electronic Raman scattering in isotropic superconductors due to long-wavelength oscillations of the electron density was considered for the first time by Abrikosov and Falkovsky [1]. They showed that at zero temperature scattering is possible only if the energy transferred to the electron system by the light exceeds the threshold frequency $\omega = 2\Delta$ (where Δ is the superconducting energy gap). This is a consequence of the absence of quasiparticle excitations at $T = 0$. Hence the only interaction of light with the bulk material is the breaking of Cooper pairs.

If one takes into account collective oscillations [2, 3] due to coupled fluctuations of the order parameter phase and electron density, then the oscillations of the latter are screened out as pointed out in reference [4]. In this case the scattering is reduced while the threshold frequency remains unchanged.

However, experimental data on light scattering in high-T_c superconductors indicate the existence of electron states within the gap, and the dependence of the maximum of the cross-section on the light polarization [5–8]. On the basis of this dependence, several authors (see, for example, [9]) have made assumptions about the symmetry of the order parameter. Additionally, the anisotropy of the order parameter explains the absence of a clear threshold [9–12].

Sometimes the two-band model of superconductivity is applied to interpret the polarization dependence [6]. The theory of this model was proposed long before the discovery of HTSC [13, 14]. An interesting feature of two-band superconductors is the existence of additional—comparing to the one-band model—collective oscillations, which correspond to small fluctuations of the relative phase of the superconducting order parameters, and of the difference of the electron densities of the two bands [15]. The system oscillates in such a way that the total electron density at every spatial point of the superconductor remains constant. The frequency of these oscillations, ω, is finite while the wave vector, $\boldsymbol{k}$, tends to zero, and can become less than the energy gaps of the superconductor. This can change the threshold frequency of the Raman effect.

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The aim of this paper is to study the Raman light scattering on the basis of the two-band model of superconductors [13–15], taking into account collective oscillations, polarization corrections required because of the long-wavelength Coulomb interaction, and scattering of electrons by non-magnetic impurities. The influence of impurity in the one-band case is studied in [16].

2. Basic equations

The equilibrium problem is described by the two-band Hamiltonian [13, 14, 17], and the order parameters (or energy gaps) of a two-band superconductor in the equilibrium state satisfy the following system of equations:

$$\begin{aligned}\frac{1}{\lambda_{11}} &= \int_{\Delta_1}^{\omega_D} \frac{\tanh x/2T}{\sqrt{x^2-\Delta_1^2}}\,\mathrm{d}x + \frac{\lambda_{12}}{\lambda_{11}}\frac{\Delta_2}{\Delta_1}\int_{\Delta_2}^{\omega_D} \frac{\tanh x/2T}{\sqrt{x^2-\Delta_2^2}}\,\mathrm{d}x \\ \frac{1}{\lambda_{22}} &= \int_{\Delta_2}^{\omega_D} \frac{\tanh x/2T}{\sqrt{x^2-\Delta_2^2}}\,\mathrm{d}x + \frac{\lambda_{21}}{\lambda_{22}}\frac{\Delta_1}{\Delta_2}\int_{\Delta_1}^{\omega_D} \frac{\tanh x/2T}{\sqrt{x^2-\Delta_1^2}}\,\mathrm{d}x\end{aligned} \tag{1}$$

where $\lambda_{nm} = V_{nm}N_m$, N_n and Δ_n are the density of electron states on the Fermi surface for one spin direction and the order parameter for the nth band, and the V_{nm} are the interaction constants (with $n = m$ for intra-band interaction, and $n \neq m$ for inter-band interaction).

Light scattering can be described by the differential cross-section in terms of the solid angle $\mathrm{d}\Omega$ and the frequency interval $\mathrm{d}\nu$:

$$\begin{aligned}\frac{\partial^2\sigma}{\partial\nu\,\partial\Omega} &= \frac{\omega_S}{\omega_I}r_0^2 S_{\gamma\gamma}(\boldsymbol{k},\nu) \\ S_{\gamma\gamma}(\boldsymbol{k},\nu) &= -\frac{1}{\pi}(1+n(\nu))\,\mathrm{Im}\,\chi_{\gamma\gamma}(\boldsymbol{k},\nu).\end{aligned} \tag{2}$$

Here $r_0 = e^2/mc^2$ is Thompson's radius, ω_I and ω_S are the frequencies of the incident and scattered light (we take $\hbar = k_B = 1$), $\nu = \omega_I - \omega_S$, $n(\nu)$ is the Bose–Einstein distribution function, and $\chi_{\gamma\gamma}$ is the Raman response function which differs from the usual density–density response function for a given band through the vertex $\gamma_n(\boldsymbol{p})$ defined as follows [4]:

$$\gamma_n(\boldsymbol{p}) = m\sum_{\alpha,\beta} e_\alpha^S \frac{\partial^2\varepsilon_n(\boldsymbol{p})}{\partial p_\alpha\,\partial p_\beta} e_\beta^I \tag{3}$$

where $\boldsymbol{e}^I$ and $\boldsymbol{e}^S$ are the polarization vectors of the incident and scattered light, and $\varepsilon_n(\boldsymbol{p})$ is the dispersion law for the nth band. In our case the value $\chi_{\gamma\gamma}$ describes the total contribution of the two bands, and is obtained as a proportionality factor of the Raman fluctuations and the value $H_A = r_0|\boldsymbol{A}^I||\boldsymbol{A}^S|$ which, with the weight $\gamma_n(\boldsymbol{p})$, plays the role of an external perturbation: $H_{int}^{(n)} = \gamma_n(\boldsymbol{p})H_A$. ($\boldsymbol{A}^I$ and $\boldsymbol{A}^S$ are the vector potentials of the incident and scattered light.)

$$\delta n_{\gamma\gamma}(\boldsymbol{k},\nu) = 2\sum_{\boldsymbol{p}}\sum_{n=1,2}\gamma_n(\boldsymbol{p})\,\delta n_p(\boldsymbol{k},\nu) = -\chi_{\gamma\gamma}(\boldsymbol{k},\nu)H_A \tag{4}$$

and in order to determine $\delta n_{\gamma\gamma}$ the linear-in-H_A correction to the zero-quasiparticle distribution function must be calculated. For this purpose we make use of the method of reference [18], which allows us to study the influence of fluctuational effects and impurity on the Raman scattering in a rather simple form. In the terms of reference [2], this means that we take into account the dependence of all Green functions (including vertex corrections) on the impurity. Compared to reference [16], here we additionally consider the fluctuations

of the scalar potential and order parameter phase, i.e. we take into account loop diagrams with different numbers of superconducting vertices.

Generalizing the method described above to the two-band case [19], we derive a system of equations for linearized Green's functions integrated over the energy:

$$\begin{aligned}
&\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}_n G'_n - \omega_+\widehat{\tau}_z G'_n - \omega G'_n\widehat{\tau}_z + (-\mathrm{i}\hat{\Delta}'_n + \mathrm{i}e\varphi + \mathrm{i}H^{(n)}_{int} + \mathrm{i}e\boldsymbol{v}\cdot\boldsymbol{A}'\hat{\tau}_z + \mathrm{i}\Sigma'_n)G_n(\omega)\\
&\quad - G_n(\omega_+)(-\mathrm{i}\hat{\Delta}'_n + \mathrm{i}e\varphi + \mathrm{i}H^{(n)}_{int} + \mathrm{i}e\boldsymbol{v}\cdot\boldsymbol{A}'\hat{\tau}_z + \mathrm{i}\Sigma'_n)\\
&\quad + (-\mathrm{i}\hat{\Delta}_n + \mathrm{i}\Sigma_n(\omega_+))G'_n - G'_n(-\mathrm{i}\hat{\Delta}_n + \mathrm{i}\Sigma_n(\omega)) = 0. \qquad (5)
\end{aligned}$$

Here, $\omega_+ = \omega + \omega_0$, $\omega = (2n+1)\pi T$, $\omega_0 = 2\pi nT$, and $\boldsymbol{v}_n$ is the electron velocity of the nth cavity of the Fermi surface. The matrix structure of the Green functions and order parameters is written as in [18]:

$$\begin{aligned}
G_n(\omega) &= \begin{pmatrix} \alpha^{(n)} & -\mathrm{i}\beta^{(n)} \\ \mathrm{i}\beta^{(n)} & -\alpha^{(n)} \end{pmatrix} \qquad G'_n = \begin{pmatrix} g_1^{(n)} & F_1^{(n)} \\ -F_2^{(n)} & g_2^{(n)} \end{pmatrix}\\
\hat{\Delta}_n &= \begin{pmatrix} 0 & \Delta_n \\ -\Delta_n & 0 \end{pmatrix} \qquad \hat{\Delta}'_n = \begin{pmatrix} 0 & \Delta'^{(n)}_1 \\ -\Delta'^{(n)}_2 & 0 \end{pmatrix}
\end{aligned} \qquad (6)$$

where

$$\begin{aligned}
\alpha^{(n)}(\omega) &= \omega\Big/\sqrt{\omega^2+\Delta_n^2} \qquad \beta^{(n)}(\omega) = \Delta_n\Big/\sqrt{\omega^2+\Delta_n^2}\\
\Sigma_n(\omega) &= \frac{1}{2\tau_n}G_n(\omega) \qquad \Sigma'_n = \frac{1}{2\tau_n}\langle G'_n\rangle
\end{aligned}$$

where angle brackets denote averaging over the solid angle of momentum $\boldsymbol{p}$ on the Fermi surface.

The linearized Green functions satisfy the normalization condition

$$G'_n G_n(\omega) + G_n(\omega_+)G'_n = 0 \qquad (7)$$

which allows one to express non-diagonal corrections to the Green functions $F_1^{(n)}$ and $F_2^{(n)}$ through diagonal ones, $g_1^{(n)}$ and $g_2^{(n)}$. From (5) and (7) we obtain corrections that we shall need later as follows:

$$\begin{aligned}
g_1^{(n)} + g_2^{(n)} &= \frac{1}{\tau_n}\left(B_n + \frac{1}{\tau_n}\right)\frac{\langle g_1^{(n)} + g_2^{(n)}\rangle}{W_n}\\
&\quad + \frac{\omega_0(\alpha_+^{(n)} - \alpha^{(n)})}{B_n W_n}\left(B_n + \frac{1}{\tau_n}\right)\left[2\mathrm{i}e\varphi - \frac{(\beta_+^{(n)} + \beta^{(n)})(\Delta'^{(n)}_1 - \Delta'^{(n)}_2)}{\alpha_+^{(n)} - \alpha^{(n)}}\right]\\
&\quad + \frac{\omega_0(\alpha_+^{(n)} - \alpha^{(n)})}{B_n W_n}\left(B_n + \frac{1}{\tau_n}\right)2\mathrm{i}H_A\gamma_n + 2\mathrm{i}e(\boldsymbol{v}\cdot\boldsymbol{A}')(\alpha_+^{(n)} - \alpha^{(n)})\frac{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}_n}{W_n} \qquad (8)
\end{aligned}$$

where

$$W_n = (B_n + 1/\tau_n)^2 + (\boldsymbol{k}\cdot\boldsymbol{v}_n)^2$$

and

$$B_n = \sqrt{\omega^2+\Delta_n^2} + \sqrt{(\omega+\omega_0)^2+\Delta_n^2}$$

and where $\boldsymbol{A}'$ and φ are the vector and scalar potentials arising in the superconductor due to the external perturbation H_A. The impurity, as in [18], is considered in the Born approximation. τ_n is the electron relaxation time in the nth band, and $\tau_n^{-1} = 2\pi N_n|V|^2 n_{kp}$ (V is the matrix element for electron–impurity scattering, and n_{kp} is the impurity concentration). Inter-band scattering is not considered, i.e. the intra-band relaxation

time is assumed to be much less than the inter-band one. Also it is assumed that the order parameters in both bands are isotropic, and that the attraction between electrons (both intra- and inter-band) is constant. Electron–hole symmetry about the Fermi surface is also assumed, and, therefore, electron density fluctuations are coupled with order parameter phase fluctuations and are not coupled with the fluctuations of the order parameter amplitude. Moreover, even in the case of electron–hole asymmetry, when the fluctuations of the phase *and* amplitude of the order parameter are coupled [20], amplitude modes for small transfer vectors $\boldsymbol{k}$ do not contribute to the light scattering.

The distribution function for the Raman fluctuations in the approach linear in H_A and the scalar potential φ can be written as follows:

$$-\delta n_{\gamma\gamma}(\boldsymbol{k}, \mathrm{i}\omega_0) = \left\{ 2 \sum_{n=1,2} \langle \gamma_n N_n (H_A \gamma_n + e\varphi) \rangle + \mathrm{i}\pi T \sum_{\omega} \sum_{n=1,2} N_n \langle \gamma_n (g_1^{(n)} + g_2^{(n)}) \rangle \right\}. \qquad (9)$$

For $\gamma_n = 1$, $H_A = 0$, and $n = 1$, equation (9) gives the well-known formula which determines the variations of the particle density [21].

The first term in (9) describes the variation of the electron density associated with the superconducting pair condensate, and the second one is due to quasiparticle fluctuations (electron-like and hole-like). In (9) the following generalizations are made: two bands with different weights of Raman fluctuations are taken into consideration, and $e\varphi$ is replaced by $e\varphi + H_A\gamma_n$, because the term $r_0 |\boldsymbol{A}^I||\boldsymbol{A}^S|\gamma_n$ enters the single-particle Hamiltonian in a similar way to $e\varphi$.

The functions $g_1^{(n)} + g_2^{(n)}$ depend on the fluctuations of the order parameter phase, the scalar potential φ, and the vector potential $\boldsymbol{A}'$. The latter need not be considered as shown below. The values $\Delta_1'^{(n)} - \Delta_2'^{(n)}$ and $e\varphi$ describe the deviation from the equilibrium theory of the two-band model [13, 14] due to the interaction between electrons, which was not taken into account in the self-consistent-field approach. They can be determined from a system of three equations. Two of them are derived from the self-consistent condition. One of them can be written as

$$\frac{1}{\lambda_{11}} (\Delta_1'^{(1)} - \Delta_2'^{(1)}) + \pi T \sum_{\omega} \frac{\beta_+^{(1)} + \beta^{(1)}}{\alpha_+^{(1)} - \alpha^{(1)}} \langle g_1^{(1)} + g_2^{(1)} \rangle + \frac{\lambda_{12}}{\lambda_{11}} \pi T \sum_{\omega} \frac{\beta_+^{(2)} + \beta^{(2)}}{\alpha_+^{(2)} - \alpha^{(2)}} \langle g_1^{(2)} + g_2^{(2)} \rangle = 0 \qquad (10)$$

and the second can be obtained by swapping the band indices: $1 \leftrightarrow 2$

The third equation is Poisson's equation, which takes into account the change of the electron density caused by light, and the two-band character of the system:

$$k^2 \varphi = -8\pi e \sum_{n=1,2} N_n \left[e\varphi + \bar{\gamma}_n H_A + \frac{\mathrm{i}\pi T}{2} \sum_{\omega} \langle g_1^{(2)} + g_2^{(2)} \rangle \right]. \qquad (11)$$

The last term in (8) is proportional to the vector potential $\boldsymbol{A}'$ which, like the scalar one, must be determined from Maxwell's equations. However, due to the gauge invariance of equations (5), a gauge condition can be selected which leaves only a transverse component of $\boldsymbol{A}'$. Then, since $\boldsymbol{k} \cdot \boldsymbol{A}' = 0$, the average of the term including $\boldsymbol{A}'$ over the solid angle of $\boldsymbol{p}$ is zero (if the other parameters remain isotropic). Therefore, from now on we will consider only the scalar potential φ, which is determined from Poisson's equation. As to the gauge invariance of the theory, note that the continuity equation results from the equation for the Green's functions if one takes into account (10) and (11).

The value $\langle\gamma_n(g_1^{(n)}+g_2^{(n)})\rangle$ must be obtained in order to determine the value $\delta n_{\gamma\gamma}(\boldsymbol{k},\nu)$. It can easily be derived from equation (8), averaging it with the weighting γ_n. Substituting the resulting average into (9), we have

$$-\delta n_{\gamma\gamma}(\boldsymbol{k},\mathrm{i}\omega_0)=2H_A\sum_{n=1,2}N_n\left\{\overline{\gamma}_n^2+\pi T\sum_{\omega}\frac{\omega_0(\alpha^{(n)}-\alpha_+^{(n)})}{B_n}\left(B_n+\frac{1}{\tau_n}\right)\left\langle\frac{\gamma_n^2}{W_n}\right\rangle\right.$$
$$\left.+\pi T\sum_{\omega}\frac{\omega_0(\alpha^{(n)}-\alpha_+^{(n)})}{B_n}\frac{(B_n+\tau_n^{-1})^2}{W_1^{(n)}}\frac{1}{\tau_n}\left\langle\frac{\gamma_n}{W_n}\right\rangle^2\right\}$$
$$+2e\varphi\sum_{n=1,2}N_n(\overline{\gamma}_n+\widetilde{Q}_{33}^{(n)})-\mathrm{i}\sum_{n=1,2}N_n\widetilde{Q}_{23}^{(n)}(\Delta'^{(n)}_1-\Delta'^{(n)}_2) \tag{12}$$

where

$$\widetilde{Q}_{23}^{(n)}=\pi T\sum_{\omega}(\beta_+^{(n)}+\beta^{(n)})\frac{\omega_0}{B_n}\frac{(B_n+\tau_n^{-1})}{W_1^{(n)}}\left\langle\frac{\gamma_n}{W_n}\right\rangle \tag{13}$$

$$\widetilde{Q}_{33}^{(n)}=\pi T\sum_{\omega}\frac{\omega_0(\alpha^{(n)}-\alpha_+^{(n)})}{B_n}\frac{(B_n+\tau_n^{-1})}{W_1^{(n)}}\left\langle\frac{\gamma_n}{W_n}\right\rangle \tag{14}$$

$$W_1^{(n)}=1-\frac{1}{\tau_n}\left(B_n+\frac{1}{\tau_n}\right)\left\langle\frac{1}{W_n}\right\rangle.$$

The fluctuational corrections in (12) can be obtained from the system (10), (11).

3. Fluctuational corrections

Substituting $\langle g_1^{(n)}+g_2^{(n)}\rangle$ into (10) and (11) we derive a system of three inhomogeneous equations to determine $\Delta'^{(1)}_1-\Delta'^{(1)}_2$, $\Delta'^{(2)}_1-\Delta'^{(2)}_2$, and φ:

$$Q\begin{pmatrix}\Delta'^{(1)}_1-\Delta'^{(1)}_2\\ \Delta'^{(2)}_1-\Delta'^{(2)}_2\\ 2\mathrm{i}e\varphi\end{pmatrix}=\begin{pmatrix}M_1\\M_2\\M_3\end{pmatrix} \tag{15}$$

where

$$Q=\begin{pmatrix}Q_{22}^{(1)}+\dfrac{\lambda_{12}}{\lambda_{11}}\dfrac{\Delta_2}{\Delta_1}I_2 & \dfrac{\lambda_{12}}{\lambda_{11}}(Q_{22}^{(2)}-I_2) & Q_{23}^{(1)}+\dfrac{\lambda_{12}}{\lambda_{11}}Q_{23}^{(2)}\\ \dfrac{\lambda_{21}}{\lambda_{22}}(Q_{22}^{(1)}-I_1) & Q_{22}^{(2)}+\dfrac{\lambda_{21}}{\lambda_{22}}\dfrac{\Delta_1}{\Delta_2}I_1 & Q_{23}^{(2)}+\dfrac{\lambda_{21}}{\lambda_{22}}Q_{23}^{(1)}\\ N_1Q_{23}^{(1)}/(N_1+N_2) & N_2Q_{23}^{(2)}/(N_1+N_2) & Q_{33}\end{pmatrix} \tag{16}$$

where

$$I_n=\int_{\Delta_n}^{\omega_D}\frac{\mathrm{d}x}{\sqrt{x^2-\Delta_n^2}}$$

and

$$Q_{33}=1+\frac{k^2}{8\pi e^2(N_1+N_2)}+\frac{N_1Q_{33}^{(1)}+N_2Q_{33}^{(2)}}{(N_1+N_2)}$$
$$Q_{22}^{(n)}=\pi T\sum_{\omega}\left(\frac{\beta_+^{(n)}+\beta^{(n)}}{2\Delta_n}+\frac{(\beta_+^{(n)}+\beta^{(n)})^2}{\alpha^{(n)}-\alpha_+^{(n)}}\frac{\omega_0}{B_n}\frac{(B_n+\tau_n^{-1})}{W_1^{(n)}}\left\langle\frac{1}{W_n}\right\rangle\right). \tag{17}$$

$Q_{23}^{(n)}$ and $Q_{33}^{(n)}$ differ from $\widetilde{Q}_{23}^{(n)}$ and $\widetilde{Q}_{33}^{(n)}$ by the replacement $\langle\gamma_n/W_n\rangle\to\langle1/W_n\rangle$. The self-consistency conditions (1) for Δ_1 and Δ_2 were used while obtaining the elements of the matrix Q.

The right-hand side of (15) is

$$M_1 = -2\mathrm{i}H_A\left[\widetilde{Q}_{23}^{(1)} + \frac{\lambda_{12}}{\lambda_{11}}\widetilde{Q}_{23}^{(2)}\right]$$
$$M_2 = -2\mathrm{i}H_A\left[\widetilde{Q}_{23}^{(2)} + \frac{\lambda_{21}}{\lambda_{22}}\widetilde{Q}_{23}^{(1)}\right] \tag{18}$$
$$M_3 = -2\mathrm{i}H_A(N_1+N_2)^{-1}[N_1(\overline{\gamma}_1 + \widetilde{Q}_{33}^{(1)}) + N_2(\overline{\gamma}_2 + \widetilde{Q}_{33}^{(2)})].$$

It follows from (15) that

$$\Delta_1'^{(1)} - \Delta_2'^{(1)} = \frac{Q_1}{Q} \qquad \Delta_1'^{(2)} - \Delta_2'^{(2)} = \frac{Q_2}{Q} \qquad 2\mathrm{i}e\varphi = \frac{Q_3}{Q} \tag{19}$$

where Q_1, Q_2, and Q_3 can be obtained by replacing the ith column in Q by the right-hand side of equation (15).

4. The clean limit

We now consider zero temperature. Using the method of analytical continuation $\mathrm{i}\omega_0 \to \nu + \mathrm{i}\delta$ $(\delta \to +0)$ [22], the expressions (13), (14), and (17) can be calculated. As a result, for the clean limit $(\tau_n^{-1} = 0)$ we obtain

$$\widetilde{Q}_{23}^{(n)} = -\mathrm{i}\nu\left\langle \frac{\gamma_n f_{\boldsymbol{k}}^{(n)}}{2\Delta_n}\right\rangle$$
$$\widetilde{Q}_{33}^{(n)} = \nu^2\left\langle \frac{\gamma_n(f_{\boldsymbol{k}}^{(n)}-1)}{\nu^2 - (\boldsymbol{k}\cdot\boldsymbol{v})^2}\right\rangle \tag{20}$$
$$Q_{22}^{(n)} = -\left\langle \frac{\nu^2 - (\boldsymbol{k}\cdot\boldsymbol{v})^2}{4\Delta_n^2} f_{\boldsymbol{k}}^{(n)}\right\rangle.$$

Here, the $f_{\boldsymbol{k}}^{(n)}$ are the well-known functions

$$f_{\boldsymbol{k}}^{(n)} = \frac{1}{\mathrm{i}\beta_n}\frac{1}{\sqrt{1-(\beta_n)^2}}\ln(\mathrm{i}\beta_n + \sqrt{1-(\beta_n)^2}) \tag{21}$$

where $\beta_n^2 = (\nu^2 - (\boldsymbol{k}\cdot\boldsymbol{v})^2)/4\Delta_n^2$.

The values $Q_{23}^{(n)}$ and $Q_{33}^{(n)}$ can be obtained replacing γ_n by 1 in (20). Now we know all of the values that we need to calculate Q (see (16)). Its zeros determine the spectrum of collective oscillations. Keeping the terms up to the order of $k^2/(8\pi e^2(N_1+N_2))$ and k^2v^2/Δ^2, we have

$$Q = Q_L D$$

where

$$D = \frac{1}{4\Delta_1\Delta_2(N_1+N_2)}\left(\frac{k^2\nu^2}{8\pi e^2} - \frac{1}{3}N_1k^2v_1^2 - \frac{1}{3}N_2k^2v_2^2\right)$$
$$Q_L = \frac{f_{\boldsymbol{k}}^{(1)}f_{\boldsymbol{k}}^{(2)}\nu^2 a}{4\Delta_1\Delta_2} - \frac{\lambda_{21}f_{\boldsymbol{k}}^{(1)} + \lambda_{12}f_{\boldsymbol{k}}^{(2)}}{\lambda_{11}\lambda_{22}}$$
$$a = 1 - -\frac{\lambda_{21}\lambda_{12}}{\lambda_{11}\lambda_{22}}.$$

Thus Q contains two factors. From the condition $D = 0$ we obtain the frequency of the plasma oscillations for the two-band model [23].

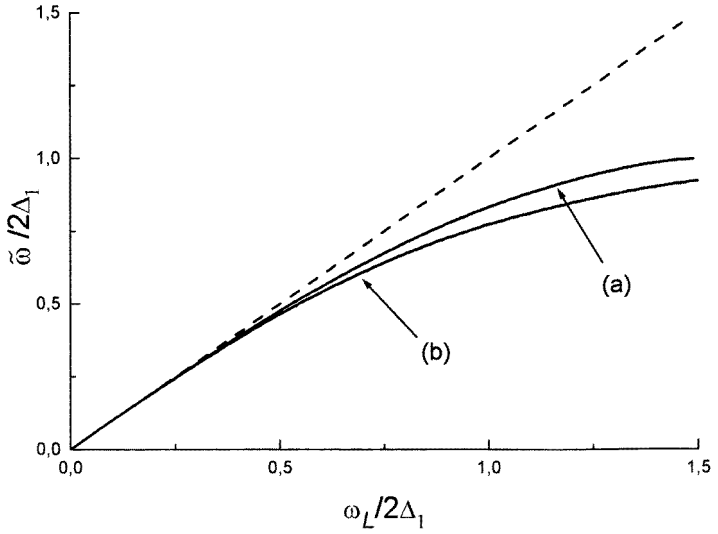

Figure 1. The frequency of the collective mode due to oscillations of the relative phase of the order parameters of the two bands versus the value $\omega_L/2\Delta_1$. The order parameters selected are as follows: (a) $\Delta_1/\Delta_2 = 0.9$ and (b) $\Delta_1/\Delta_2 = 0.25$. The relation of the densities of states for the two curves is $N_1/N_2 = 0.8$. The dashed line corresponds to the case where the frequency coincides with Leggett's frequency.

When $\nu^2/4\Delta_n^2 \ll 1$ the values $f_{\boldsymbol{k}}^{(1)}$ are equal to 1, and from $Q = 0$ we have

$$\omega_L^2 = \frac{\lambda_{12}(1 + N_1/N_2)\, 4\Delta_1\Delta_2}{\lambda_{11}\lambda_{22} - \lambda_{21}\lambda_{12}}. \tag{22}$$

Here the relation $\lambda_{12}/\lambda_{21} = N_2/N_1$ was used. ω_L is the frequency of the oscillations caused by the fluctuations of the difference of the electron densities of the two bands, which was obtained for the first time by Leggett [15].

The value Q is of the order of k^2; therefore, the values Q_1, Q_2, and Q_3 (formula (19)) which determine the fluctuational contribution must be expanded up to the same order. In this manner we obtain the fluctuational contribution, and substituting it into (12) and then using the definition of $\chi_{\gamma\gamma}$, equation (4), we obtain

$$\chi_{\gamma\gamma}(\boldsymbol{k} \to 0, \nu) = \sum_{n=1,2} \chi_{\gamma\gamma}^{nn}(\boldsymbol{k} \to 0, \nu) + \chi_{\gamma\gamma}^{12}(\boldsymbol{k} \to 0, \nu) \tag{23}$$

where

$$\chi_{\gamma\gamma}^{nn}(\boldsymbol{k} \to 0, \nu) = 2N_n\left(\langle f_{\boldsymbol{k}}^{(n)}\gamma_n^2\rangle - \frac{\langle f_{\boldsymbol{k}}^{(n)}\gamma_n\rangle^2}{\langle f_{\boldsymbol{k}}^{(n)}\rangle}\right)$$

$$\chi_{\gamma\gamma}^{12}(\boldsymbol{k} \to 0, \nu) = -\frac{2N_1\lambda_{12}}{\lambda_{11}\lambda_{22}}\frac{1}{Q_L}\left(\langle f_{\boldsymbol{k}}^{(1)}\gamma_1\rangle\sqrt{\frac{\langle f_{\boldsymbol{k}}^{(2)}\rangle}{\langle f_{\boldsymbol{k}}^{(1)}\rangle}} - \langle f_{\boldsymbol{k}}^{(2)}\gamma_2\rangle\sqrt{\frac{\langle f_{\boldsymbol{k}}^{(1)}\rangle}{\langle f_{\boldsymbol{k}}^{(2)}\rangle}}\right).$$

Thus, the fluctuational contribution leads to screening of the Raman scattering [4] and to the appearance of an additional term independent of $\boldsymbol{k}$ caused by inter-band transitions of Cooper pairs. The next terms in (23) not taken into account are of the order of

$$\frac{k^2}{8\pi e^2(N_1 + N_2)}\frac{k^2v^2}{\Delta^2}D^{-1} \ll 1.$$

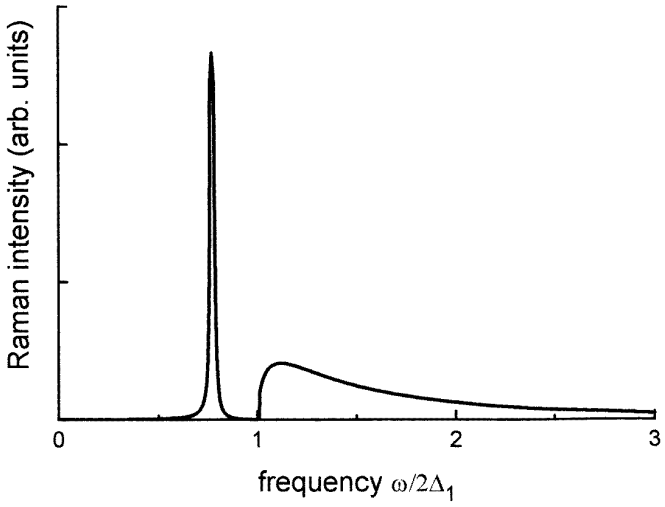

Figure 2. The fluctuational contribution to the Raman light scattering caused by inter-band transitions of Cooper pairs. The interaction constants are: $\lambda_{11} = 0.3, \lambda_{22} = 0.05, \lambda_{12} = 0.01$, and $\lambda_{21} = 0.008$. The relation of the densities of states is $N_1/N_2 = 0.8$. This gives $\Delta_1/\Delta_2 = 0.25$.

The first term in (23) is an additive contribution from the two bands. $\operatorname{Im}\chi^{nn}_{\gamma\gamma}(\boldsymbol{k}, \nu)$ is non-zero only at frequencies $\nu > 2\Delta_n$. Let us write the value γ_n as a sum of isotropic and anisotropic parts: $\gamma_n = \gamma_n^0 + \delta\gamma_n(\boldsymbol{p})$. At $\gamma_n = \gamma_n^0$ the oscillations of the electron density are screened completely, and $\operatorname{Im}\chi_{\gamma\gamma}(\boldsymbol{k} \to 0, \nu) = 0$. This satisfies the particle number conservation law [9]. Thus at constant $\gamma_n(\boldsymbol{p})$ and for $\boldsymbol{k} \to 0$, only inter-band interactions of Cooper pairs contribute to the light scattering. The value $\operatorname{Im}\chi^{12}_{\gamma\gamma}(\boldsymbol{k} \to 0, \nu)$ differs from zero in the case where γ_1 and γ_2 are constant but not equal. For this case we have

$$\operatorname{Im}\chi^{12}_{\gamma\gamma}(\boldsymbol{k} \to 0, \nu) = \frac{2\lambda_{12}\lambda_{21}}{(\lambda_{11}\lambda_{22})^2}(\gamma_1^0 - \gamma_2^0)^2|Q_L|^{-2}[N_1 \operatorname{Im} f^{(2)}_{\boldsymbol{k}}|f^{(1)}_{\boldsymbol{k}}|^2 + N_2 \operatorname{Im} f^{(1)}_{\boldsymbol{k}}|f^{(2)}_{\boldsymbol{k}}|^2] + \frac{2N_1\lambda_{12}}{(\lambda_{11}\lambda_{22})^2}(\gamma_1^0 - \gamma_2^0)^2\delta(\nu - \widetilde{\omega})\pi P_L. \tag{24}$$

Here $\widetilde{\omega}$ is determined as a root of the equation $Q_L = 0$, and P_L is the pole's weight:

$$P_L^{-1} = \left.\frac{\mathrm{d}Q_L}{\mathrm{d}\nu}\right|_{\nu=\widetilde{\omega}}.$$

For $\nu/2\Delta_1 \ll 1$ and $\nu/2\Delta_2 \ll 1$ the frequency $\widetilde{\omega}$ coincides with Leggett's frequency [15]: $\widetilde{\omega} = \omega_L$.

The dependence of $\widetilde{\omega}$ on the parameters of the two-band model is presented in figure 1. The superconducting order parameters Δ_1 and Δ_2 have been calculated self-consistently using equations (1). It is evident from the figure that $\widetilde{\omega}$ coincides with ω_L only at small frequencies $\nu/2\Delta_1 \leqslant 0,5$. However, over the whole interval of frequencies where there is a solution of $Q_L = 0$, it is less than the energy gap: $\widetilde{\omega}/2\Delta_1 < 1$.

The intensity of the Raman scattering calculated using equation (24) is presented in figures 2 and 3. The sharp peaks on the figures correspond to the frequency $\widetilde{\omega}$. A similar peak in the Raman intensity was predicted by Wu and Griffin [24] for a model of a layered superconductor with tunnelling of Cooper pairs between layers. Continuum scattering at

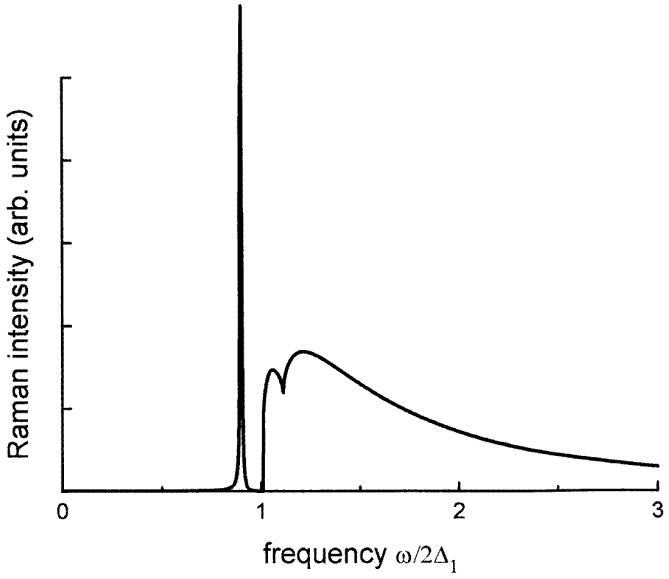

Figure 3. The same plot as in figure 2, but for $\Delta_1/\Delta_2 = 0.8$ ($\lambda_{11} = 0.045$, $\lambda_{22} = 0.055$, $\lambda_{12} = 0.08$, $\lambda_{21} = 0.066$, and $N_1/N_2 = 0.4$).

$\nu > 2\Delta_1$ corresponds to the first term in (24). It essentially depends on the relations of the density of states N_1/N_2 and the order parameters Δ_1/Δ_2 of the two bands. It can be seen that if Δ_1 strongly differs from Δ_2, then the continuum scattering intensity has only one maximum, close to the lowest of the two gaps. In the case where $\Delta_1/\Delta_2 \simeq 1$, there are two maxima in the plot.

In reference [6] the electron Raman scattering for $YBa_2Cu_4O_8$ was found to depend on the light polarization. At different polarizations, the scattering intensity decreases below different frequencies, which was interpreted by the authors as implying the existence of two different energy gaps. This conclusion was confirmed by the results from phonon Raman scattering. The proposed two-gap interpretation [6] corresponds to two contributions of $\chi^{nn}_{\gamma\gamma}$ (formula (23)): at different polarizations, different terms $\chi^{nn}_{\gamma\gamma}$ are seen.

The absence of a clear threshold is explained by the authors of [6] in terms of an anisotropy of the order parameters. Note that, although our calculations are for the isotropic case, the results for $\chi^{nn}_{\gamma\gamma}$ can be applied for anisotropic gaps. The contribution to the Raman effect caused by the inter-band transitions of Cooper pairs ($\chi^{12}_{\gamma\gamma}$) for anisotropic order parameters has a more complicated form. However, at $\gamma_1^0 \neq \gamma_2^0$, as well as in the isotropic case, this contribution exists for any polarization, and begins from the lowest energy gap. The data of reference [6] do not support this. It is possible that the two-band nature observed in [6] is unconventional, or that the contribution of $\mathrm{Im}\,\chi^{12}_{\gamma\gamma}$ is not observed due to λ_{12} and λ_{21} being small. Equation (1) can have a solution with $\Delta_2/\Delta_1 \simeq 3$ (as in [6]) at small λ_{12} with one of the inter-band constants equal to zero (either λ_{11} or λ_{22}). This is the case where superconductivity in one of the bands exists only due to transitions of Cooper pairs from another band. In this case $\widetilde{\omega} = 0$, and no δ-like peak is seen, as was also the case in [6]. Note that in reference [25] the following estimates for the order parameters of yttrium ceramics (supposed therein to be two-band superconductors) were used: $\lambda_{12} \gg \lambda_{21}, \lambda_{22} \simeq 0, \lambda_{21} \simeq 0,4, \lambda_{11} \simeq 2,5$. So, comparing our results to the

experimental data of [6], we can conclude that if the superconductivity in yttrium ceramics has a traditional two-band nature [6], then in one of the bands it is induced.

5. The impure case

Similarly to in the clean limit, for the case of a superconductor containing non-magnetic impurities it is easy to obtain the response function $\widetilde{\chi}_{\gamma\gamma}(\boldsymbol{k}\to 0,\nu)$ as a sum of two terms:

$$\widetilde{\chi}_{\gamma\gamma}(\boldsymbol{k}\to 0,\nu)=\sum_{n=1,2}\widetilde{\chi}^{nn}_{\gamma\gamma}(\boldsymbol{k}\to 0,\nu)+\widetilde{\chi}^{12}_{\gamma\gamma}(\boldsymbol{k}\to 0,\nu). \tag{25}$$

Here

$$\widetilde{\chi}^{nn}_{\gamma\gamma}(\boldsymbol{k}\to 0,\mathrm{i}\omega_0)=2N_n(\overline{\gamma_n^2}-\overline{\gamma}_n^2)\pi T\sum_{\omega}\frac{\omega_0(\alpha^{(n)}-\alpha^{(n)}_+)}{B_n(B_n+\tau_n^{-1})} \tag{26}$$

$$\widetilde{\chi}^{12}_{\gamma\gamma}(\boldsymbol{k}\to 0,\nu)=2\sum_{n=1,2}N_n\left\{(1+Q^{(n)}_{33})\overline{\gamma}_n^2 + \frac{1}{H_A}\,e\varphi(\overline{\gamma}_n+\tilde{Q}^{(n)}_{33})-\frac{\mathrm{i}}{2H_A}\widetilde{Q}^{(n)}_{23}(\Delta_1'^{(n)}-\Delta_2'^{(n)})\right\}. \tag{27}$$

Formulae (26) and (27) are obtained without restrictions on the values $\nu\tau_n$ and $\Delta_n\tau_n$. From (26) in the case of one band the result of reference [16] for $\mathrm{Im}\,\chi_{\gamma\gamma}$ follows. In the two-band model, from (26) we have an additional contribution, which, for instance, is in the case of normal metal

$$\mathrm{Im}\,\widetilde{\chi}^{nn}_{\gamma\gamma}=\sum_{n=1,2}N_n(\overline{\gamma_n^2}-\overline{\gamma}_n^2)\frac{2\nu}{\tau_n(\nu^2+\tau_n^{-2})}.$$

To calculate $\chi^{12}_{\gamma\gamma}$, taking the limit $\boldsymbol{k}\cdot\boldsymbol{v}=0$ is not appropriate, and we have to keep terms up to the order of k^2v^2. To simplify the problem we consider the case of large $1/\tau_n$, i.e. where $\tau_n^{-1}\gg\Delta$ and $\tau_n^{-1}\gg\nu$. Then, for $k^2v^2\ll 1$ the small parameter $k^2v^2\tau/\Delta$ appears, and all values can be expanded in terms of this parameter. The fluctuational contribution in the dirty limit proved to be equal to that of the clean limit:

$$\mathrm{Im}\,\widetilde{\chi}_{12}(\boldsymbol{k}\to 0,\nu)=\mathrm{Im}\,\chi_{12}(\boldsymbol{k}\to 0,\nu) \tag{28}$$

and can be determined from (24). The frequency of the collective mode ω_L is given by (22). For the plasma frequency we have

$$\omega_{pl}^2=\frac{8\pi^2e^2}{3}(N_1v_1^2\tau_1\Delta_1+N_2v_2^2\tau_2\Delta_2). \tag{29}$$

Plasma oscillations influence the Raman scattering only in terms of the order of $k^2v^2\tau/\Delta\ll 1$.

The inter-band electron–impurity scattering times (not considered in the present work) are assumed to satisfy the inequality $\tau_{12}^{-1}\ll\Delta$. In the opposite limit, it is necessary to take them into account, and the problem is reduced to the one-band model with renormalized parameters [26, 27].

6. Conclusion

So, we have considered electron Raman scattering in two-band superconductors, taking into account collective oscillations, Coulomb screening, and scattering by the non-magnetic impurities.

We have shown that for a clean superconductor and for a superconductor with non-magnetic impurities, the intensity of the Raman scattering contains terms that are diagonal and terms that are non-diagonal in the band index. The diagonal ones describe additive contributions from the two energy bands. Fluctuational effects screen these terms in the clean limit as well as in the dirty limit. If the wave vector transferred to the bulk material by the light is small, and the vertices γ_n are independent of the solid angle of $\boldsymbol{p}$, then the electron density oscillations are screened completely, and the diagonal terms are equal to zero. Also, fluctuational effects cause a non-diagonal contribution to the Raman scattering. These terms are possible for any polarization; they are not connected with fluctuations of the electron density, and hence are not screened. The non-diagonal terms contain a sharp peak inside the lowest gap and a continuum above it, whose shape depends on the choice of parameters of the two-band model. However, in all cases the lowest gap serves as the threshold of a non-diagonal continuum spectrum. Experimentally, this means that the lowest gap must be active for any light polarization.

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